

# A remark on the existence of solutions to a $(k_1, k_2)$ -Hessian system with convection term

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## Abstract

In this paper, two new results and short proofs are given for the existence of positive entire large and bounded radial positive solutions for the following nonlinear system with gradient term

$$\begin{cases} S_{k_1}(\lambda(D^2 u_1)) + b_1(|x|)|\nabla u_1|^{k_1} = p_1(|x|)f_1(u_1, u_2) \text{ for } x \in \mathbb{R}^N, \\ S_{k_2}(\lambda(D^2 u_2)) + b_2(|x|)|\nabla u_2|^{k_2} = p_2(|x|)f_2(u_1, u_2) \text{ for } x \in \mathbb{R}^N, \end{cases}$$

where  $S_{k_i}(\lambda(D^2 u_i))$  is the  $k_i$ -Hessian operator,  $b_1, p_1, f_1, b_2, p_2$  and  $f_2$  are continuous functions satisfying certain properties. Our results complete and improve a recently work published by Zhang and Zhou. The main difficulty in dealing with our system is the presence of the convection term.

2010 AMS Subject Classification: Primary:35J25, 35J47 Secondary: 35J96.

Key words. Entire solution; Large solution; Elliptic system

## 1 Introduction

The purpose of this paper is to present new results concerning the nonlinear Hessian system with convection term

$$\begin{cases} S_{k_1}(\lambda(D^2 u_1)) + b_1(|x|)|\nabla u_1|^{k_1} = p_1(|x|)f_1(u_1, u_2) \text{ for } x \in \mathbb{R}^N \ (N \geq 3), \\ S_{k_2}(\lambda(D^2 u_2)) + b_2(|x|)|\nabla u_2|^{k_2} = p_2(|x|)f_2(u_1, u_2) \text{ for } x \in \mathbb{R}^N \ (N \geq 3), \end{cases} \quad (1)$$

where  $b_1, p_1, f_1, b_2, p_2, f_2$  are continuous functions satisfying certain properties,  $k_1, k_2 \in \{1, 2, \dots, N\}$  and  $S_{k_i}(\lambda(D^2 u_i))$  stands for the  $k_i$ -Hessian operator defined as the sum of all  $k_i \times k_i$  principal minors of the Hessian matrix  $D^2 u_i$ . For instance, the following well known operators are included in this class:

Operator:	Laplacian	Monge–Ampère
	$S_1(\lambda(D^2 u_i)) = \Delta u_i = \operatorname{div}(\nabla u_i)$	$S_N(\lambda(D^2 u_i)) = \det(D^2 u_i)$

In recent years equations of the type (1) have been the subject of rather deep investigations since appears from many branches of mathematics and applied mathematics. For more surveys on these questions we advise the paper of Alves and Holanda [1], Bao-Ji and Li [3], Bandle and Giarrusso [2], Cirstea and Rădulescu [4], Clément-Manásevich and Mitidieri [5], De Figueiredo and Jianfu [6], Galaktionov and Vázquez [7], Ghergu and Rădulescu [8], Salani [19], Ji and Bao [12], Jian [11], Peterson and Wood [16], Quittner [17], Li and Yang [13], Li-Zhang and Zhang [14], Viaclovsky [20, 21] and not the last Zhang and Zhou [22].

The motivation for studying (1) comes from the work of Ghergu and Rădulescu [8] where in discussion is a system of the type

$$\begin{cases} \Delta u_1 + |\nabla u_1| = p_1(|x|) f_1(u_1, u_2) & \text{for } x \in \mathbb{R}^N \ (N \geq 3), \\ \Delta u_2 + |\nabla u_2| = p_2(|x|) f_2(u_1, u_2) & \text{for } x \in \mathbb{R}^N \ (N \geq 3), \end{cases}$$

and from the recently work of Zhang and Zhou [22] where the authors have been considered the system

$$\begin{cases} S_k(\lambda(D^2 u_1)) = p_1(|x|) f_1(u_2) & \text{for } x \in \mathbb{R}^N \ (N \geq 3), \\ S_k(\lambda(D^2 u_2)) = p_2(|x|) f_2(u_1) & \text{for } x \in \mathbb{R}^N \ (N \geq 3). \end{cases}$$

To simplify the presentation we wish to mention that our purpose is to complete and improve all the results in [22] for the more general system (1). By analogy with the work of Zhang and Zhou [22] we introduce the following notations

$$\begin{aligned} C_0 &= (N-1)!/[k_1!(N-k_1)!], C_{00} = (N-1)!/[k_2!(N-k_2)!], \\ B_1^-(\xi) &= \frac{\xi^{k_1-N}}{C_0} e^{-\int_0^\xi \frac{1}{C_0} t^{k_1-1} b_1(t) dt}, B_1^+(\xi) = \xi^{N-1} e^{\int_0^\xi \frac{1}{C_0} t^{k_1-1} b_1(t) dt} p_1(\xi), \\ B_2^-(\xi) &= \frac{\xi^{k_2-N}}{C_{00}} e^{-\int_0^\xi \frac{1}{C_{00}} t^{k_2-1} b_2(t) dt}, B_2^+(\xi) = \xi^{N-1} e^{\int_0^\xi \frac{1}{C_{00}} t^{k_2-1} b_2(t) dt} p_2(\xi), \\ P_1(r) &= \int_0^r \left( B_1^-(r) \int_0^r B_1^+(t) dt \right)^{\frac{1}{k_1}} dr, \\ P_2(r) &= \int_0^r \left( B_2^-(r) \int_0^r B_2^+(t) dt \right)^{\frac{1}{k_2}} dr, \\ F_{1,2}(r) &= \int_{a_1+a_2}^r \frac{1}{f_1^{1/k_1}(t, t) + f_2^{1/k_2}(t, t)} dt \text{ for } r \geq a_1 + a_2 > 0, a_1 \geq 0, a_2 \geq 0, \\ P_1(\infty) &= \lim_{r \rightarrow \infty} P_1(r), P_2(\infty) = \lim_{r \rightarrow \infty} P_2(r), F_{1,2}(\infty) = \lim_{s \rightarrow \infty} F_{1,2}(s). \end{aligned}$$

We will always assume that the variable weights functions  $b_1, b_2, p_1, p_2$  and the nonlinearities  $f_1, f_2$  satisfy:

(P1)  $b_1, b_2 : [0, \infty) \rightarrow [0, \infty)$  and  $p_1, p_2 : [0, \infty) \rightarrow [0, \infty)$  are spherically symmetric continuous functions (i.e.,  $p_i(x) = p_i(|x|)$  and  $b_i(x) = b_i(|x|)$  for  $i = 1, 2$ );

(C1)  $f_1, f_2 : [0, \infty) \times [0, \infty) \rightarrow [0, \infty)$  are continuous and increasing.

Here is a first (somewhat suprizing) result:

**Theorem 1** *We assume that  $F_{1,2}(\infty) = \infty$  and (P1), hold. Furthermore, if  $f_1$  and  $f_2$  satisfy (C1), then the system (1) has at least one positive radial solution  $(u_1, u_2) \in C^2([0, \infty)) \times C^2([0, \infty))$  with central value in  $(a_1, a_2)$ . Moreover, the following hold:*

- 1.) *If  $P_1(\infty) + P_2(\infty) < \infty$  then  $\lim_{r \rightarrow \infty} u_1(r) < \infty$  and  $\lim_{r \rightarrow \infty} u_2(r) < \infty$ .*
- 2.) *If  $P_1(\infty) = \infty$  and  $P_2(\infty) = \infty$  then  $\lim_{r \rightarrow \infty} u_1(r) = \infty$  and  $\lim_{r \rightarrow \infty} u_2(r) = \infty$ .*

In the same spirit we also have, our next result:

**Theorem 2** Assume that the hypotheses (P1) and (C1) are satisfied. Then, if  $F_{1,2}(\infty) < \infty$ ,  $P_1(\infty) + P_2(\infty) < \infty$  and  $P_1(\infty) + P_2(\infty) < F_{1,2}(\infty)$  the system (1) has one positive bounded radial solution  $(u_1, u_2) \in C^2([0, \infty)) \times C^2([0, \infty))$ , with central value in  $(a_1, a_2)$ , such that

$$\begin{cases} a_1 + f_1^{1/k_1}(a_1, a_2) P_1(r) \leq u_1(r) \leq F_{1,2}^{-1}(P_1(r) + P_2(r)), \\ a_2 + f_2^{1/k_2}(a_1, a_2) P_2(r) \leq u_2(r) \leq F_{1,2}^{-1}(P_1(r) + P_2(r)). \end{cases}$$

## 2 Proofs of the main results

We give in this section the proof of Theorem 1 and Theorem 2. For the readers' convenience, we recall the radial form of the  $k$ -Hessian operator.

**Remark 1** If  $u : \mathbb{R}^N \rightarrow \mathbb{R}$  is radially symmetric then a calculation show

$$S_k(\lambda(D^2u(r))) = r^{1-N} C_{N-1}^{k-1} \left[ \frac{r^{N-k}}{k} (u'(r))^k \right]',$$

where the prime denotes differentiation with respect to  $r = |x|$  and  $C_{N-1}^{k-1} = (N-1)! / [(k-1)!(N-k)!]$ .

This Remark is well known, see for example [12] or [19].

**Proof of the Theorems 1 and 2.** We start by showing that the system (1) has positive radial solutions. For this purpose, we show that the ordinary differential equations system

$$\begin{cases} C_{N-1}^{k_1-1} \left[ \frac{r^{N-k_1}}{k_1} e^{\int_0^r \frac{1}{C_0} t^{k_1-1} b_1(t) dt} (u_1'(r))^{k_1} \right]' = r^{N-1} e^{\int_0^r \frac{1}{C_0} t^{k_1-1} b_1(t) dt} p_1(r) f_1(u_1(r), u_2(r)) \text{ for } r > 0, \\ C_{N-1}^{k_2-1} \left[ \frac{r^{N-k_2}}{k_2} e^{\int_0^r \frac{1}{C_{00}} t^{k_2-1} b_2(t) dt} (u_2'(r))^{k_2} \right]' = r^{N-1} e^{\int_0^r \frac{1}{C_{00}} t^{k_2-1} b_2(t) dt} p_2(r) f_2(u_1(r), u_2(r)) \text{ for } r > 0, \\ u_1'(r) \geq 0 \text{ and } u_2'(r) \geq 0 \text{ for } r \in [0, \infty), \\ u_1(0) = a_1 \text{ and } u_2(0) = a_2, \end{cases} \quad (2)$$

has solution. Therefore, at least one solution of (2) can be obtained using successive approximation by defining the sequences  $\{u_1^m\}^{m \geq 1}$  and  $\{u_2^m\}^{m \geq 1}$  on  $[0, \infty)$  in the following way  $u_1^0 = a_1$ ,  $u_2^0 = a_2$  for  $r \geq 0$  and

$$\begin{cases} u_1^m(s) = a_1 + \int_0^r \left[ B_1^-(t) \int_0^t B_1^+(s) f_1(u_1^{m-1}(s), u_2^{m-1}(s)) ds \right]^{1/k_1} dt, \\ u_2^m(s) = a_2 + \int_0^r \left[ B_2^-(t) \int_0^t B_2^+(s) f_2(u_1^{m-1}(s), u_2^{m-1}(s)) ds \right]^{1/k_2} dt. \end{cases} \quad (3)$$

It is easy to see that  $\{u_1^m\}^{m \geq 1}$  and  $\{u_2^m\}^{m \geq 1}$  are non-decreasing on  $[0, \infty)$ . Indeed, we consider

$$\begin{aligned} u_1^1(r) &= a_1 + \int_0^r \left[ B_1^-(t) \int_0^t B_1^+(s) f_1(u_1^0(s), u_2^0(s)) ds \right]^{1/k_1} dt \\ &= a_1 + \int_0^r \left[ B_1^-(t) \int_0^t B_1^+(s) f_1(a_1, a_2) ds \right]^{1/k_1} dt \\ &\leq a_1 + \int_0^r \left[ B_1^-(t) \int_0^t B_1^+(s) f_1(u_1^1(s), u_2^1(s)) ds \right]^{1/k_1} dt = u_1^2(r). \end{aligned}$$

This implies that

$$u_1^1(r) \leq u_1^2(r) \text{ which further produces } u_1^2(r) \leq u_1^3(r).$$

Continuing, an induction argument applied to (3) show that for any  $r \geq 0$  we have

$$u_1^m(r) \leq u_1^{m+1}(r) \text{ and } u_2^m(r) \leq u_2^{m+1}(r) \text{ for any } m \in \mathbb{N}$$

i.e.,  $\{u_1^m\}^{m \geq 1}$  and  $\{u_2^m\}_{j=1,2}^{m \geq 1}$  are non-decreasing on  $[0, \infty)$ . By the monotonicity of  $\{u_1^m\}^{m \geq 1}$  and  $\{u_2^m\}^{m \geq 1}$  we have the inequalities

$$C_{N-1}^{k_1-1} \left\{ \frac{r^{N-k_1}}{k_1} e^{\int_0^r \frac{1}{C_0} t^{k_1-1} b_1(t) dt} [(u_1^m(r))']^{k_1} \right\}' \leq B_1^+(r) f_1(u_1^m(r), u_2^m(r)), \quad (4)$$

$$C_{N-1}^{k_2-1} \left\{ \frac{r^{N-k_2}}{k_2} e^{\int_0^r \frac{1}{C_{00}} t^{k_2-1} b_2(t) dt} [(u_2^m(r))']^{k_2} \right\}' \leq B_2^+(r) f_2(u_1^m(r), u_2^m(r)). \quad (5)$$

After integration from 0 to  $r$ , an easy calculation yields

$$\begin{aligned} (u_1^m(r))' &\leq \left( B_1^-(r) \int_0^r B_1^+(t) f_1(u_1^m(t), u_2^m(t)) dt \right)^{\frac{1}{k_1}} \\ &\leq \left( B_1^-(r) \int_0^r B_1^+(t) f_1(u_1^m(t) + u_2^m(t), u_1^m(t) + u_2^m(t)) dt \right)^{\frac{1}{k_1}} \\ &\leq \left( f_1^{1/k_1} + f_2^{1/k_2} \right) (u_1^m(r) + u_2^m(r), u_1^m(r) + u_2^m(r)) \left( B_1^-(r) \int_0^r B_1^+(t) dt \right)^{\frac{1}{k_1}}. \end{aligned} \quad (6)$$

As before, exactly the same type of conclusion holds for  $(u_2^m(r))'$ :

$$\begin{aligned} (u_2^m(r))' &\leq \left( B_2^-(r) \int_0^r B_2^+(z) f_2(u_1^m(z), u_2^m(z)) dz \right)^{1/k_2} \\ &\leq \left( f_1^{1/k_1} + f_2^{1/k_2} \right) (u_1^m(r) + u_2^m(r), u_1^m(r) + u_2^m(r)) \left( B_2^-(r) \int_0^r B_2^+(t) dt \right)^{\frac{1}{k_2}}. \end{aligned} \quad (7)$$

Summing the two previous inequalities (6) and (7), we obtain

$$\frac{(u_1^m(r) + u_2^m(r))'}{\left( f_1^{1/k_1} + f_2^{1/k_2} \right) (u_1^m(r) + u_2^m(r), u_1^m(r) + u_2^m(r))} \leq \left( B_1^-(r) \int_0^r B_1^+(t) dt \right)^{\frac{1}{k_1}} + \left( B_2^-(r) \int_0^r B_2^+(t) dt \right)^{\frac{1}{k_2}}. \quad (8)$$

Integrating from 0 to  $r$  the inequality (8), we obtain

$$\int_{a_1+a_2}^{u_1^m(r)+u_2^m(r)} \frac{1}{f_1^{1/k_1}(t, t) + f_2^{1/k_2}(t, t)} dt \leq P_1(r) + P_2(r).$$

We now have

$$F_{1,2}(u_1^m(r) + u_2^m(r)) \leq P_1(r) + P_2(r), \quad (9)$$

which will play a basic role in the proof of our main results. The inequalities (9) can be reformulated as

$$u_1^m(r) + u_2^m(r) \leq F_{1,2}^{-1}(P_1(r) + P_2(r)). \quad (10)$$

This can be easily seen from the fact that  $F_{1,2}$  is a bijection with the inverse function  $F_{1,2}^{-1}$  strictly increasing on  $[0, \infty)$ . So, we have found upper bounds for  $\{u_1^m\}^{m \geq 1}$  and  $\{u_2^m\}^{m \geq 1}$  which are dependent of  $r$ . We are now ready to give a complete proof of the Theorems 1-2.

**Proof of Theorem 1 completed:** When  $F_{1,2}(\infty) = \infty$  it follows that the sequences  $\{u_j^m\}_{j=1,2}^{m \geq 1}$  are bounded and equicontinuous on  $[0, c_0]$  for arbitrary  $c_0 > 0$ . Possibly after passing to a subsequence, we may assume

that  $\{u^m\}_{j=1,2}^{m \geq 1}$  converges uniformly to  $\{u_j\}_{j=1,2}$  on  $[0, c_0]$ . At the end of this process, we conclude by the arbitrariness of  $c_0 > 0$ , that  $(u_1, u_2)$  is positive entire solution of system (2). The solution constructed in this way will be radially symmetric. Since the radial solutions of the ordinary differential equations system (2) are solutions (1) it follows that the radial solutions of (1) with  $u_1(0) = a_1$ ,  $u_2(0) = a_2$  satisfy:

$$u_1(r) = a_1 + \int_0^r \left( B_1^-(y) \int_0^y B_1^+(t) f_1(u_1(t), u_2(t)) dt \right)^{1/k_1} dy, \quad r \geq 0, \quad (11)$$

$$u_2(r) = a_2 + \int_0^r \left( B_2^-(y) \int_0^y B_2^+(t) f_2(u_1(t), u_2(t)) dt \right)^{1/k_2} dy, \quad r \geq 0. \quad (12)$$

Next, it is easy to verify that the Cases 1. and 2. occur.

**Case 1.):** When  $P_1(\infty) + P_2(\infty) < \infty$ , it is not difficult to deduce from (11) and (12) that

$$u_1(r) + u_2(r) \leq F_{1,2}^{-1}(P_1(\infty) + P_2(\infty)) < \infty \text{ for all } r \geq 0,$$

and so  $(u_1, u_2)$  is bounded. We next consider:

**Case 2.):** In the case  $P_1(\infty) = P_2(\infty) = \infty$ , we observe that

$$\begin{aligned} u_1(r) &= a_1 + \int_0^r \left( B_1^-(t) \int_0^t B_1^+(s) f_1(u_1(s), u_2(s)) ds \right)^{\frac{1}{k_1}} dt \\ &\geq a_1 + f_1^{1/k_1}(a_1, a_2) \int_0^r \left( B_1^-(t) \int_0^t B_1^+(s) ds \right)^{\frac{1}{k_1}} dt \\ &= a_1 + f_1^{1/k_1}(a_1, a_2) P_1(r). \end{aligned} \quad (13)$$

The same computations as in (13) yields

$$u_2(r) \geq a_2 + f_2^{1/k_2}(a_1, a_2) P_2(r).$$

and passing to the limit as  $r \rightarrow \infty$  in (13) and in the above inequality we conclude that

$$\lim_{r \rightarrow \infty} u_1(r) = \lim_{r \rightarrow \infty} u_2(r) = \infty,$$

which yields the result. We now turn to:

**Proof of Theorem 2 completed:**

In view of the above analysis, the proof can be easily deduced from

$$F_{1,2}(u_1^m(r) + u_2^m(r)) \leq P_1(\infty) + P_2(\infty) < F_{1,2}(\infty) < \infty,$$

Indeed, since  $F_{1,2}^{-1}$  is strictly increasing on  $[0, \infty)$ , we find that

$$u_1^m(r) + u_2^m(r) \leq F_{1,2}^{-1}(P_1(\infty) + P_2(\infty)) < \infty,$$

and then the non-decreasing sequences  $\{u_1^m(r)\}^{m \geq 1}$  and  $\{u_2^m(r)\}^{m \geq 1}$  are bounded above for all  $r \geq 0$  and all  $m$ . The final step, is to conclude that  $(u_1^m(r), u_2^m(r)) \rightarrow (u_1(r), u_2(r))$  as  $m \rightarrow \infty$  and the limit functions  $u_1$  and  $u_2$  are positive entire bounded radial solutions of system (1).

**Remark 2** Make the same assumptions as in Theorem 1 or Theorem 2 on  $b_1, p_1, f_1, b_2, p_2, f_2$ . If, in addition,

$$p_1(|x|) \geq \left( C_{N-1}^{k_1-1} \frac{N-k_1}{k_1} |x|^{-N} - b_1(|x|) |x|^{k_1-N} \right) \int_0^{|x|} \frac{s^{N-1}}{C_0} p_1(s) ds \text{ for every } x \in \mathbb{R}^N, \quad (14)$$

$$p_2(|x|) \geq \left( C_{N-1}^{k_2-1} \frac{N-k_2}{k_2} |x|^{-N} - b_2(|x|) |x|^{k_2-N} \right) \int_0^{|x|} \frac{s^{N-1}}{C_{00}} p_2(s) ds \text{ for every } x \in \mathbb{R}^N, \quad (15)$$

then, the solution  $(u_1, u_2)$  is convex.

**Proof.** It is clear that

$$C_{N-1}^{k_1-1} \left[ \frac{r^{N-k_1}}{k_1} e^{\int_0^r \frac{1}{C_0} t^{k_1-1} b_1(t) dt} \left( u_1'(r) \right)^{k_1} \right]' = r^{N-1} e^{\int_0^r \frac{1}{C_0} t^{k_1-1} b_1(t) dt} p_1(r) f_1(u_1(r), u_2(r)), \quad (16)$$

and integrating from 0 to  $r$  yields

$$\begin{aligned} r^{N-k_1} e^{\int_0^r \frac{1}{C_0} t^{k_1-1} b_1(t) dt} \left( u_1'(r) \right)^{k_1} &= \int_0^r \frac{s^{N-1}}{C_0} e^{\int_0^s \frac{1}{C_0} t^{k_1-1} b_1(t) dt} p_1(s) f_1(u_1(s), u_2(s)) ds \\ &\leq f_1(u_1(r), u_2(r)) \int_0^r \frac{s^{N-1}}{C_0} e^{\int_0^s \frac{1}{C_0} t^{k_1-1} b_1(t) dt} p_1(s) ds, \end{aligned}$$

which produces

$$\begin{aligned} \left( \frac{u_1'(r)}{r} \right)^{k_1} &\leq f_1(u_1(r), u_2(r)) r^{-N} e^{-\int_0^r \frac{1}{C_0} t^{k_1-1} b_1(t) dt} \int_0^r \frac{s^{N-1}}{C_0} e^{\int_0^s \frac{1}{C_0} t^{k_1-1} b_1(t) dt} p_1(s) ds \\ &\leq f_1(u_1(r), u_2(r)) r^{-N} \int_0^r \frac{s^{N-1}}{C_0} p_1(s) ds. \end{aligned} \quad (17)$$

On the other hand the inequality (16) can be written in the following way

$$C_{N-1}^{k_1-1} u_1''(r) \left( \frac{u_1'(r)}{r} \right)^{k_1-1} + C_{N-1}^{k_1-1} \frac{N-k_1}{k_1} \left( \frac{u_1'(r)}{r} \right)^{k_1} + b_1(r) \left( u_1'(r) \right)^{k_1} = p_1(r) f_1(u_1(r), u_2(r)). \quad (18)$$

Using the inequality (17) in (18) we obtain

$$\begin{aligned} p_1(r) f_1(u_1(r), u_2(r)) &\leq C_{N-1}^{k_1-1} u_1''(r) \left( \frac{u_1'(r)}{r} \right)^{k_1-1} + C_{N-1}^{k_1-1} \frac{N-k_1}{k_1} f_1(u_1(r), u_2(r)) r^{-N} \int_0^r \frac{s^{N-1}}{C_0} p_1(s) ds \\ &\quad + b_1(r) r^{k_1} f_1(u_1(r), u_2(r)) r^{-N} \int_0^r \frac{s^{N-1}}{C_0} p_1(s) ds, \end{aligned}$$

from which we have

$$f_1(u_1(r), u_2(r)) \left[ p_1(r) - \left( C_{N-1}^{k_1-1} \frac{N-k_1}{k_1} r^{-N} - b_1(r) r^{k_1-N} \right) \int_0^r \frac{s^{N-1}}{C_0} p_1(s) ds \right] \leq C_{N-1}^{k_1-1} u_1''(r) \left( \frac{u_1'(r)}{r} \right)^{k_1-1},$$

which completes the proof of  $u_1''(r) \geq 0$ . A similar argument produces  $u_2''(r) \geq 0$ . We also remark that, in the simple case  $b_1 = b_2 = 0$ ,  $s^{N-1} p_1(s)$  and  $s^{N-1} p_2(s)$  are increasing then (14) and (15) hold.

**Acknowledgement.** The author would like to thank to the editors and reviewers for valuable comments and suggestions which contributed to improve this article.

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